Int. J. Heat Mass Transfer. Vol. 36, No. 3, pp. 645–650, 1993
Printed in Great Britain

0017–9310/93 $6.00 + 0.00
Pergamon Press Ltd

An application of spectral decomposition to model validation in the thermal analysis of buildings

Y. CANDAU and G. PIAR
L.E.T.I.E.F. URA1508, Université Paris XII, 94000 Creteil, France

(*Received* 8 *November* 1991 *and in final form* 24 *March* 1992)

Abstract—We compare building thermal behaviour as obtained from experiment and from a physical model. Experimental data are analysed by use of an identified model, decomposed as a sum of first-order models. The physical model is reduced to an equivalent form by use of its eigenelements. This method provides an intrinsic basis for comparison. Analysis showed that the dynamic behaviour of the building could only be explained by using a two-dimensional model for conduction in one of the walls; the agreement between experimental data and theoretical model was then quite good.

INTRODUCTION

VALIDATION is always a crucial step in the development of physical models; this is especially true in building physics, with the recent trend to develop huge modular simulation tools, and the need for reliable elementary models. Still, validation is often made on an *ad hoc* basis, giving no precise idea of the real discrepancy between model and measure, and little insight as to the cause of this discrepancy.

In this paper, we use model identification and spectral analysis to analyse the content of, respectively, experimental data and theoretical models. This approach provides an intrinsic basis for comparison and can be applied to any linear or linearized model; it can also give some indication as to how an incorrect model should be modified.

Model identification techniques have been of constant use in automatics or in process engineering but their introduction in building thermics is relatively recent; theoretical [1, 2] as well as experimental [3, 4] work has shown that the global thermal behaviour of a building can be reasonably represented by low-order linear models. Model identification is generally used to obtain operative models; here we contend that it is also an efficient way to explore the informational content of experimental data, in a context of linear systems theory.

Spectral decomposition is a classic way to analyse and solve sets of linear differential equations through diagonalization of the system: as such sets of equations appear naturally in many domains of physics by discretization of partial differential equations [5], it is not surprising that the method has been widely used and commented upon, especially in the area of vibration analysis. It has also been used in solving heat transfer problems (such methods as integral transform or variables separation are closely related [6]); the application to complex systems such as buildings is more recent [7, 8].

Here, the model to be analysed relates the temperature inside a building to externally imposed temperature and heat flux conditions, or excitations. Experimental data were obtained on real-size experimental building cells.

THE TEST CELLS

The E.T.N.A. test cells are a facility of the Research Centre of Electricité de France at Les Renardieres [9]

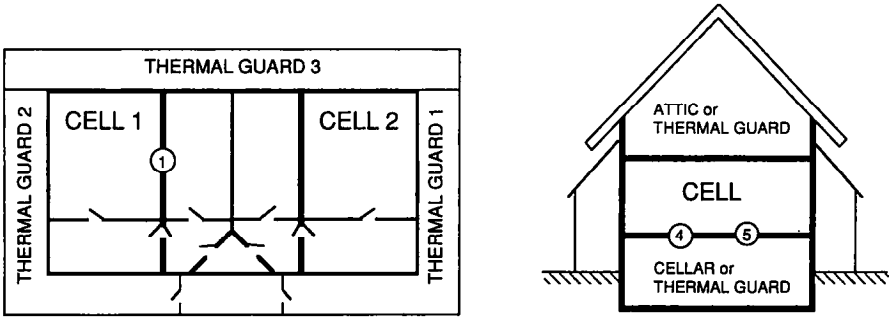

FIG. 1. The E.T.N.A. test cells. Numbered walls are referred to in the text.

NOMENCLATURE

A	dynamic matrix	T_e	temperature outside building [°C]
B	excitation matrix	U	static heat loss coefficient
E	vector of excitations	z	variable in z-transform
C	matrix of thermal capacities	z^{-1}	delay operator.
G	static gain of identified model		
h	transfer function		
H	matrix of transfer functions	Greek symbols	
n	eigenvector	β	static gain of theoretical model
N	matrix of eigenvectors	Δ	time step
P	heating power [kW]	λ	eigenvalue
T	vector of node temperatures	Λ	diagonal matrix of eigenvalues
T_i	temperature inside building [°C]	τ	time constant.

(Fig. 1). This is a real-size building, including two identical symmetrical cells, surrounded by controlled volumes. By use of removable external partitions, the cells can be submitted to a partially or totally artifical climate. Extensive data acquisition and processing capabilities are provided.

For the present experiment, the cells were put in a totally artificial climate (particularly no solar radiation), with an electrical internal heat source. Data were taken at 10 min intervals, then digitally filtered and sampled with a time step $\Delta = 1$ h. The cell internal temperature was taken as the mean of five dry-bulb temperatures at different levels; external temperature was taken as the mean of surrounding volumes temperatures; internal heating power is the average power over the time step.

EXPERIMENTAL RESULTS AND MODEL IDENTIFICATION

The test sequence (Fig. 2), whose initial objective was to obtain a simple characterization of the test cells [10], consists of a classic step input applied to heating power; the temperature of the surrounding volumes was kept nearly constant, but due to the fact that no cooling apparatus was yet installed, it was not possible to avoid a drift of up to 2°C during the heating period. Both cells react in much the same way, and we shall subsequently refer to one of the cells only.

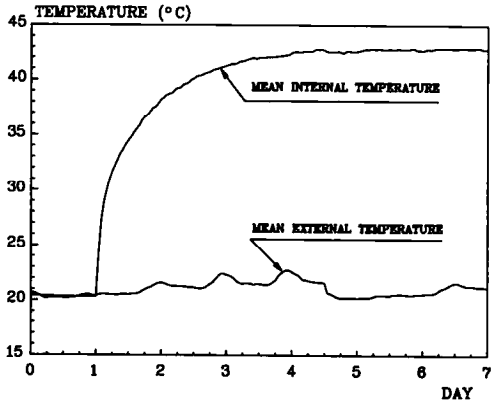

FIG. 2. Experimental temperatures.

A direct treatment of these measurements gave a static heat loss coefficient (U-value) of $U = 53.1$ W °C^{-1} and a main time constant (as obtained from a linear regression on the logarithm of reduced temperature) $\tau = 23$ h.

A more sophisticated treatment consists of using identification techniques, which enables us to obtain a more complete description of the physical system, and also to take into account minor defects in the experiment such as the external temperature drift.

The identification procedure we use is based on a sampled linear systems representation [11], using transfer functions in the variable z^{-1} to relate the internal temperature T_i to excitations, namely external temperature T_e and heating power P. The transfer functions are modelled as polynomials, the coefficients of which are determined by non-linear least squares minimization. The determination of the correct polynomial order is made through a complex decision-making process, making extensive use of auto-correlation and cross-correlation analysis and statistical tests [12]. Using this procedure implies that the form of the model will be dependent on the information content of the data, and may vary from one experiment to the other. Thus we are more interested in the physical information that can be extracted from an identified model than in its overall representativity.

For the present experiment, the identification procedure gave the following transfer function model:

$$(1-1.6304z^{-1}+0.6431z^{-2})T_i(°C)$$
$$= 0.0127T_e(°C)+(3.182-2.948z^{-1})P(\text{kW}) \quad (1)$$

with a residual mean-square error $\sigma = 0.26$°C, which shows that this model achieves a very good fit to experimental data.

Of course, given the experimental conditions, this model does not provide much information about the

dynamics of the response to external temperature, and we shall therefore concentrate on the response to heating power.

This response is characterized by the second-order transfer function

$$\frac{3.182-2.948z^{-1}}{1-1.6304z^{-1}+0.06431z^{-2}}$$

which can be decomposed as

$$\frac{2.799}{1-0.6687z^{-1}}+\frac{0.3827}{1-0.9617z^{-1}}. \tag{2}$$

Expression (2) shows that this response is the sum of two first-order systems; each of these systems can be characterized by a time constant τ and a static gain G, both parameters being readily related to the transfer function expression

$$\begin{cases} e^{-\Delta/\tau_1} = 0.9617 \Rightarrow \tau_1 = 24.9\,\text{h}; \; G_1 = 8.45\,^{\circ}\text{C kW}^{-1} \\ e^{-\Delta/\tau_2} = 0.6687 \Rightarrow \tau_2 = 2.48\,\text{h}; \; G_2 = 9.99\,^{\circ}\text{C kW}^{-1}. \end{cases}$$

We see that this analysis exhibits two very different time constants, corresponding to roughly the same proportion of the global static response. The inverse of the sum of the static gains is the global U-value for the identified model

$$U = \frac{1}{G_1+G_2} = 54.3\,\text{W}\,^{\circ}\text{C}^{-1}.$$

The above analysis could be extended to any transfer function model: an nth order model relative to a given excitation would be expressed in terms of n doublets (time constant, static gain).

This kind of representation for a linear dynamic system, which corresponds to a decomposition in elementary independent systems, is a spectral representation and provides a convenient way to summarize the behaviour of the system. It should be noted that this representation can be obtained in a straightforward fashion from a classical so-called ARMAX model; it is not necessary to set any special form for the model to be identified.

SPECTRAL ANALYSIS FOR A PHYSICAL MODEL

A typical model for heat transfer in a building [13], using finite-differences schemes for conduction through walls and linearizing the transfer equations at wall surfaces, will yield a set of ordinary differential equations

$$C\cdot\dot{T} = A\cdot T + B\cdot E \tag{3}$$

where T is a vector of temperatures at the different 'nodes', E is a vector of excitations or externally imposed conditions and B a corresponding influence matrix, C is a diagonal matrix of thermal capacities (c_i), and matrix A is symmetric (because of the reciprocity of conductive, radiative and convective heat transfer equations) and definite negative (because of the Second Law).

A classic transformation [14] is to use the square root matrix of C ($c_i^{1/2}$) to perform a change in variables

$$X = C^{1/2}\cdot T$$

which transforms the differential system into the standard form

$$\dot{X} = F\cdot X + G\cdot E \tag{4}$$

where $F = C^{-1/2}\cdot A\cdot C^{-1/2}$ and $G = C^{-1/2}\cdot B$.

The main advantage of this transformation is that it preserves the symmetry of the dynamic matrix F: the eigenvalues (λ_i) of F are thus real, negative, with corresponding eigenvectors (n_i) forming an orthonormal complete set, which means that the matrix N with ith column the ith eigenvector n_i has the property: $N^{-1} = N^{\mathrm{T}}$.

The transformation of (4) into the base of eigenvectors gives a set of decoupled first-order differential equations

$$N^{\mathrm{T}}\cdot\dot{X} = (N^{\mathrm{T}}\cdot F\cdot N)\cdot N^{\mathrm{T}}\cdot X + (N^{\mathrm{T}}\cdot G)\cdot E$$
$$\Rightarrow \dot{x} = \Lambda\cdot x + g\cdot E \tag{5}$$

where $\Lambda = (\lambda_i)$ is diagonal.

Taking the Laplace transform of (5) gives

$$p\cdot x^* = \Lambda\cdot x^* + g\cdot E^*$$
$$\Rightarrow x^* = (p\cdot I-\Lambda)^{-1}\cdot g\cdot E^* \tag{6}$$

where $(pI-\Lambda)^{-1}$ is the diagonal matrix with diagonal elements $1/(p-\lambda_i)$.

Thus, in the Laplace domain, the solution of the original system (3) is

$$T^* = C^{-1/2}\cdot X^* = C^{-1/2}\cdot N\cdot x^*$$
$$= C^{-1/2}\cdot N\cdot(p-\Lambda)^{-1}\cdot N^{\mathrm{T}}\cdot C^{-1/2}\cdot B\cdot E^*$$
$$= H\cdot E^*. \tag{7}$$

The elements of the matrix H are given by a straightforward, if tedious, calculation

$$h_{ij} = \sum_k \frac{1}{p-\lambda_k}\left(\sum_l c_i^{-1/2}c_l^{-1/2}n_{ik}n_{lk}b_{lj}\right) \tag{8}$$

which we rewrite

$$h_{ij} = \sum_k \frac{\beta_{ij}^k}{1+\tau_k p} \tag{9}$$

where

$$\begin{cases} \tau_k = -1/\lambda_k \\ \beta_{ij}^k = \tau_k\left(\sum_l c_i^{-1/2}c_l^{-1/2}n_{ik}n_{lk}b_{lj}\right); \end{cases} \tag{10}$$

h_{ij} is the transfer function between a given excitation e_j and the temperature of a given node T_i; we see that h_{ij} is the sum of first-order transfer functions, characterized each by a time constant τ_k and a static gain β_{ij}^k.

Thus, the behaviour of a system described by a set of differential equations like (3) can be described by

establishing for each excitation j a spectral map (τ_k, β_{ij}^k) of the transfer function decomposition, which gives for each temperature node i the influence of each mode k. This representation is of the same kind as the one we derived from an identified sampled linear model. Application of this analysis to the test cell described before will provide a common basis for comparison to experimental data.

THEORETICAL ANALYSIS OF THE TEST CELL

A discrete model for the test cell was obtained with the following assumptions:

(1) the air inside the cell is at a uniform temperature;

(2) transfers in walls are unidimensional and calculated through a finite differences method;

(3) convective and radiative heat transfers at wall surfaces are linearized and globalized by use of an equivalent heat transfer coefficient.

A complete description of geometry and wall characteristics can be found in ref. [15].

The global model consists of seven conductive walls of varying complexity and one air capacity.

The discretization of heat conduction equation was made in the following way: an initial repartition of nodes was empirically chosen, and the number of nodes in each wall was then doubled until the results of modal analysis exhibited no significant differences. This led to a 65 nodes model for the test cell.

Diagonalization of matrix F in equation (4) was performed by reduction to tridiagonal form and QR algorithm [16]. This provided the total set of eigenvalues and corresponding eigenvectors. The spectral maps (τ_k, β_{ij}^k) could then be computed for any given node and any specific excitation by simple operations.

To be compatible with the results of the experiment described above, we shall only analyse the results relative to the internal air node, with internal heat flux the specified excitation.

COMPARISON OF THEORETICAL AND IDENTIFIED MODELS

The spectral map for the response of internal air temperature to heating power for the model described above shows that, of all 65 modes, only a few have a significant contribution (Fig. 3); four main modes can be identified (Table 1).

Table 1. Composition of the four main modes in Fig. 3

Mode	τ (h)	β (°C kW^{-1})	Percentage of total response (%)
65	72	0.98	6
64	28	5.79	36
63	14	3.71	23
59	0.97	3.58	22

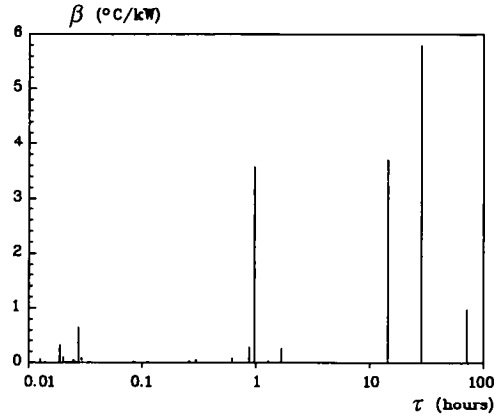

FIG. 3. Spectral map of the response to heating power.

Other modes have smaller individual contributions which add up to 13% of the total response: most of these modes correspond to very fast dynamics and could be merged into a single instantaneous response; a few other modes cluster around the 1 h values and could be regrouped with mode 59.

As such, the theoretical model exhibits some important differences with the identified model.

Visualizing the different eigenvectors can help understand the behaviour of the model; an eigenvector is a set of values, each related to a given node: if nodes are numbered in a convenient way, a plot of the eigenvector values vs node number provides a unidimensional distribution through the different elements (walls) of the model.

Analysis of the eigenvector corresponding to the 72 h value (Fig. 4) shows that this mode is mostly dependent on the presence of wall 1, which is indeed a very heavy structure (spine wall). We can admit that this mode did exist in the experiment, but was not evidenced due to a relatively short experiment duration (only twice the time constant), and also because this mode is only responsible for 6% of the total response.

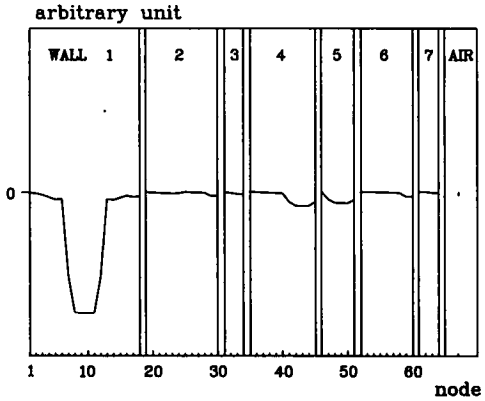

FIG. 4. Eigenvector for mode 65.

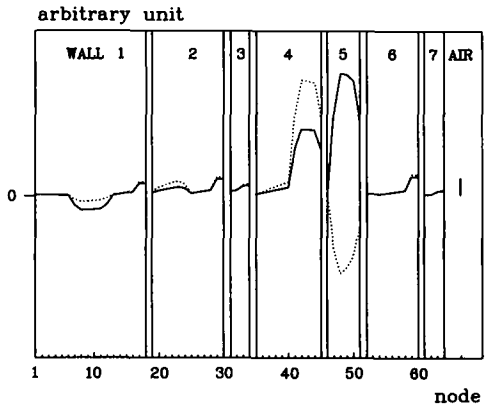

FIG. 5. Eigenvectors for modes 63 and 64.

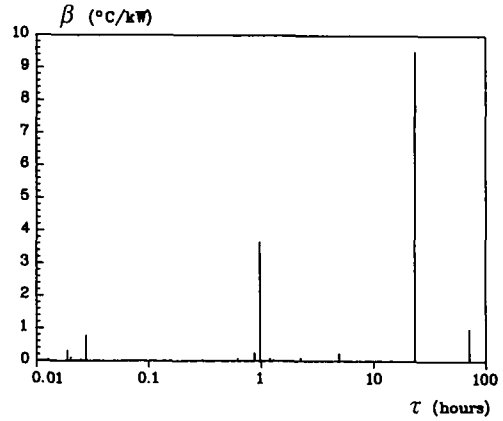

FIG. 7. Spectral map for two-dimensional model.

On the other hand, the following modes (14 and 28 h) are sufficiently important and sufficiently distinct that they should appear in the identified model.

Analysis of the corresponding eigenvectors (Fig. 5) shows that both modes are related to walls 4 and 5, with a positive coupling for the first and a negative coupling for the second; the positive-coupled mode corresponds to a situation where both walls evolve in the same way (say both walls are heating up simultaneously), while the negative-coupled mode represents a situation where the walls evolve in opposite ways (one being warmer and the other colder, and heat flux circulating from one to the other through the air node).

In this initial model, walls 4 and 5 both correspond to the floor: this is a composite structure, with a concrete slab on concrete beams, the interval between beams being filled with polystyrene (Fig. 6).

Heat transfers through the concrete beams and through the isolated part are of the same magnitude; thus it is not possible to neglect any one of these two structures, and they have been modelled as two independent unidimensional walls. In the global model these two structures are weakly coupled through the internal air node, and generate two distinct modes.

This hypothesis can be verified by removing one of the two structures from the global model: the set of corresponding modes also disappears, leaving each time only one time constant around 20 h (25 h in the first case, 17 h in the second).

In reality, the physical coupling between the two structures is much stronger and more distributed; it is logical to think that this coupling merges the two corresponding modes into one mode with intermediate value for the time constant. Also, it is clear that a distributed coupling will tend to prevent the appearance of the negative-coupled mode mentioned above.

To verify this, we made a two-dimensional model of the floor, using a classic finite-difference scheme; the equations corresponding to this model are integrated to the global model in replacement of those for walls 4 and 5, and the same analysis as above is still possible.

The spectral map for the response of the internal air node temperature to internal heat flux shows indeed that this response now consists of three main modes (Fig. 7 and Table 2).

The last mode has been previously discussed; comparison of the first two modes with those for the experimentally identified model shows a good agreement for the 'slower' mode. The rapid mode for the theoretical model has a time constant of 0.98 h instead of 2.48 h for the identified model, and, probably more important, it also has too small a static gain, even considering the total contribution of the remaining secondary modes: this means in turn that the global static heat loss coefficient for the model is higher than the measured one ($U = 0.60$ W °C^{-1}).

This last defect of the model is not rare in building thermal analysis, and is easily corrected by small changes on the conductivities of materials, or on the convective–radiative heat transfer coefficients, all values seldom known to a high degree of precision.

Unfortunately, at this point, nothing in the above analysis indicates which coefficients should be modi-

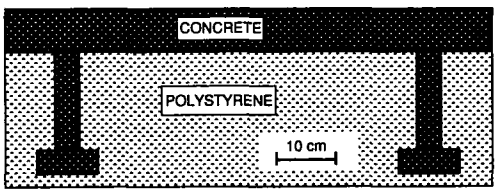

FIG. 6. Structure of the floor.

Table 2. Composition of the three main modes in Fig. 7

τ (h)	β (°C kW^{-1})	Percentage of total response (%)
23.4	9.50	58
0.98	3.66	23
71.5	0.99	6

fied: in fact, there are probably too many ways to achieve the desired correction. Also, the coupling between the different parts makes it difficult to predict in which way the system will react to a change in some element.

CONCLUSION

We have shown how spectral analysis can help us compare experimental and theoretical models; it can also help us investigate the reasons for discrepancies between experiment and theory.

Still, many problems remain to be solved: for the example presented above, we can raise at least two questions:

(1) the theoretical model presents some very fast modes and one very slow mode: do these modes exist in reality and could they be observed, or are they just artefacts due to the oversimplification of the model?

(2) while global agreement between model and reality seems good, some final adjustments remain necessary: how are these to be conducted?

The answer to the first question can only be sought experimentally, but we feel that very fast modes are introduced by the simplification of convective transfers and are not physically present; should they be, they would appear as an instantaneous response to heating (according to the model, they would represent 13% of the total response).

As to the second point, it could be suggested to perform final adjustments through identification techniques; but as mentioned above, the number of coefficients to adjust is so large that the optimal solution found this way would be meaningless: the experimental data do not contain the information necessary to determine all the parameters.

An expert kind of approach seems more promising; such an approach could be aided by such tools as sensitivity analysis: by analysing how the model reacts to changes in the parameters, some information can be gained as to its capacity to better adjust to experimental data.

Clearly, such a sensitivity analysis can be more conveniently conducted using spectral representation, and we intend to develop such tools in the near future.

Acknowledgements—The authors wish to thank EdF, Centre des Etudes et Recherches, and P. Dalicieux, for making available the experimental data presented in this paper.

REFERENCES

1. P. Bacot, A. Neveu et J. Sicard, Analyse modale des phénomènes thermiques en régime variable dans le bâtiment, *Revue Gén. Thermique* **267**, 189–201 (1984).
2. A. Neveu, P. Bacot et R. Regas, Modèles d'évolution thermique des bâtiments—Conditions pratiques d'identification, *Revue Gén. Thermique* **296–297**, 413–420 (1986).
3. R. Gicquel, F. Neirac et E. Nogaret, Identification de modèles thermiques de bâtiments, *Journées A.F.M.E. "Régulation, Commande, Télégestion"*, Sophia-Antipolis, France, 28–29 October (1987).
4. Y. Candau, H. Gharbi, M. Gonord et A. Marti, Diagnostic thermique de bâtiments: les possibilités des méthodes d'identification, *Vth Int. Meeting on Heat Transfer*, Monastir, Tunisia, 22–26 April (1991).
5. P. A. Raviart et J. M. Thomas, *Introduction à l'analyse numérique des équations aux dérivées partielles*, Chap. 6, *Théorie spectrale des problèmes aux limites*. Masson, Paris (1983).
6. M. N. Özisik, *Heat Conduction*. Wiley, New York (1980).
7. K. El Khoury et A. Neveu, Analyse modale des systèmes thermiques en présence de transferts non réciproques, *Int. J. Heat Mass Transfer* **32**, 213–226 (1989).
8. J. J. Salgon and A. Neveu, Application of modal analysis to modelling of thermal bridges in buildings, *Energy Buildings* **10**, 109–120 (1987).
9. J. Noel, Réalisation et instrumentation des cellules test en climat naturel, Note EdF HE12W2635, Electricité de France (1988).
10. P. Girault et P. Dalicieux, Caractérisation thermique des cellules ETNA, Note Edf HE12W3102, Electricité de France (1991).
11. T. Soederstroem and P. Stoica, *System Identification*. Prentice-Hall, Englewood Cliffs, New Jersey (1989).
12. H. Gharbi, Identification en thermique du bâtiment: contribution théorique et expérimentale à la détermination de paramètres physiques d'un bâtiment, Doctoral Thesis, Université Paris XII (1991).
13. Y. Candau, Thermique du bâtiment et systèmes linéaires, Doctoral Thesis, Université Paris XII (1988).
14. P. G. Ciarlet, *Analyse Numérique Matricielle et Optimisation*. Masson, Paris (1982).
15. P. Dalicieux, E. Moutaouahid et P. Girault, Contribution à la validation expérimentale de CLIM 2000 à l'aide des cellules ETNA, Note Edf HE12W3159, Electricité de France (1991).
16. W. H. Press, B. P. Flannery, S. A. Teukolsky and W. T. Vetterling, *Numerical Recipes: The Art of Scientific Computing*. Cambridge University Press, London (1989).